APPENDIX 2

MATHEMATICAL EXPRESSION ON THE METHODOLOGY OF CONSTRUCTION OF ASSOCIATED MATRICES

A2.1 This Appendix gives a brief discussion of methods of obtaining *commodity x commodity* table and *industry x industry* table under alternative technology assumptions.

A2.2 The *commodity x commodity* input-output table is suitable for multisectoral projections where final demand estimates are obtained on commodity bases. The *industry x industry* input-output table is useful in detailed planning of industries whose products include by-products also. The two alternative assumptions for transferring of outputs of secondary products are (i) industry technology assumption where input structure of a secondary product is considered to be similar to that of the industry where it has been produced and (ii) commodity technology assumption where the input structure of the secondary product of an industry is assumed to be similar to that of the industry where it is primarily produced. Besides these two main assumptions, sometimes mixed assumptions have to be followed, as all secondary products cannot just be based on only one type of technology assumption. Usually commodity technology assumption is followed for subsidiary products and industry technology assumption is appropriate for joint products and by-products.

A2.3 In a *commodity x commodity* table both rows and columns represent the commodity group sectors. If the secondary products of an industry group along-with the inputs are transferred to the industry group where they are the principal products, the resulting table is a *commodity x commodity* input-output table.

A2.4 In an *industry x industry* table, on the other hand, both rows and columns represent industry group sectors comprising of a mix of different commodity groups. The row of a sector in this table gives the supply of all products and secondary product (as a mix) produced by the corresponding industry group for different intermediate and final uses.

A2.5 The following gives briefly the methodology in mathematical terms for constructing such tables. The basic data available from industry input and output tabulations satisfy the following relationships:

Input relations: $$q_j = \sum_k x_{jk} + f_j \qquad (1)$$

Output relations: $$q_j = \sum_i m_{ij} \qquad (2)$$

$$g_i = \sum_j m_{ij} \qquad (3)$$

Where

q_j = total output of j-th commodity group
g_i = total output (of all products and by-products) of the i-th industry group
f_j = final demand of the j-th commodity
x_{jk} = output of j-th commodity used as input in the k-th sector (industry group)
m_{ij} = output of j-th commodity produced by the i-th industry group
The above symbols without subscript refer to the corresponding vectors

A2.6 A schematic arrangement of input-output data in a simplified accounting framework can be presented as follows:

	Commodities	**Industries**	**Final demand**	**Total**
Commodities		X	f	q
Industries	M			g
Primary inputs		y'		
Total	q'	g'		

A2.7 Here y denotes the column vector of y_j and y_j denotes the value of primary inputs (factor incomes) in the j-th industry. The superscript prime (') is used to denote the transpose.

A2.8 From this accounting data various other matrices can be derived further using the following notations:

A: *commodity x commodity* coefficient matrix
W: *commodity x commodity* flow matrix recording the value of purchase of commodities by commodities
B: *commodity x industry* coefficient matrix, values in the absorption matrix expressed as coefficients

$$B = X (g)^{-1}$$

C: *Product mix* matrix, columns of which show proportions in which a particular industry produces various commodities

$$C = M' (g)^{-1}$$

where g = diagonal matrix with diagonal elements as the elements of vector g

D: *Market share* matrix, the columns of which show proportions in which various industries produce the total output of a particular commodity

$$D = M (q)^{-1}$$

where q = diagonal matrix with diagonal elements as the elements of vector q

E: *industry x industry* coefficient matrix

Z: *industry x industry* flow matrix recording the value of purchases of industry outputs by industries

e: final demand for the outputs of industries.

A2.9 The derived matrices can be conveniently seen in the following schematic arrangement:

	Commodities	Industries	Final demand
Commodities	A W=Aq =BDq=BM	$B = X(g)^{-1}$	
Industries	$C = M'(g)^{-1}$ $D = M(q)^{-1}$	E Z = Eg	e

A2.10 According to the method suggested in "System of National Accounts", Studies in Method Series F.No.2, Revision 3, 1968 (UN), *commodity x commodity* table and *industry x industry* table under the two technology assumptions can be derived as under;

Commodity x commodity table:

Commodity technology: $q = (BC^{-1})\, q + f$, $W = (BC^{-1})\, q$

Industry technology: $q = (BD)\, q + f$, $W = (BD)\, q$

Industry x industry table:

Commodity technology: $g = (C^{-1} B)\, g + e$, $Z = (C^{-1}B)\, g$

Industry technology: $g = (D\, B)\, g + e$, $Z = (DB)\, g$

A2.11 For commodity technology assumption, the output or make matrix has to be square and non-singular so that inverse of the matrix could be obtained.

A2.12 In order to use *industry x industry* table under any of the assumptions, it is necessary to derive the final demand for the outputs of industries, ' e '. However, information of final demand is invariably available on commodity rather than industry basis and in order to estimate the final demand for industry outputs ' f ' has to be made industry-wise by multiplying it by the appropriate matrix. Thus under,

Commodity technology: $e = C^{-1} f$

Industry technology: $e = Df$

A2.13 The *commodity x commodity* table is found to be more suitable in most applications since demand is for a particular commodity or group of commodities and not for the mixed range of output of an industry and thus there is no need to transform the final demand vectors from one unit to another. Moreover, the calculated commodity outputs can be transformed using the market share or product mix matrix into industry output levels. This sequence of calculations makes an industry x industry table unattractive. Further, for a *commodity x commodity* table, transfers made under the commodity technology assumption, sometimes give rise to negative entries which are difficult to explain. Thus only *commodity x commodity* table under industry technology assumption has been presented in the present report.

Net indirect taxes for *commodity x commodity* table

A2.14 The input flow matrix at producers' price can be considered as the sum of two matrices, (i) the input flow at factor cost and (ii) matrix of net indirect taxes. This matrix of net indirect taxes for *commodity x industry* table is denoted by T_1.

A2.15 The matrix of net indirect taxes (T_2) for *commodity x commodity* table under the two technology assumptions can be derived in the same way as the *commodity x commodity* flow matrix is obtained from the input flow matrix as described in "Problems of input-output tables and analysis" Studies in Methods - Series F.No. 14, Revision 1, 1966 (UN). Thus under,

Commodity technology : $T_2 = T_1 (g)^{-1} C^{-1} q$
Industry technology : $T_2 = T_1 (g)^{-1} D q$

A2.16 The column totals of the net indirect tax matrix T_2 represent the total net indirect taxes on inputs consumed by various commodity groups and also on the categories of final demand. These net indirect tax totals are presented as a row at the bottom of the *commodity x commodity* table.

Value added for *commodity x commodity* table

A2.17 In National Accounts Statistics, the estimates of gross value added are prepared according to different industry groups. However, for *commodity x commodity* table the estimates of gross value added according to different commodity groups are required. In what follows, the necessary details are presented to derive the vector of gross value added for *commodity x commodity* table.

A2.18 In the notations used so far, the set of industry cost equations can be expressed in the form

$$g = y + g B' i \qquad \textbf{(4)}$$

where i denotes the unit column vector

A2.19 The set of commodity cost equations can be expressed similarly in the form

$$q = l + q A' i \qquad \textbf{(5)}$$

where l denotes the vector of gross value added corresponding to different commodity groups.

pre-multiplying (5) by D, we obtain

$$Dq = Dl + D q A' i \qquad \textbf{(6)}$$

Under **commodity technology** assumption, $A = BC^{-1}$

Also $M = Dq$ and $M' = Cg$

Thus from (6), we get

$$g = Dl + D\,q\,(C^{-1})'\,B'\,i \qquad \text{since } g = Dq$$
$$= Dl + M\,(C')^{-1}\,B'\,i$$
$$= Dl + MM^{-1}\,g\,B'\,i$$
$$= Dl + g\,B'\,i \qquad \textbf{(7)}$$

On comparing (4) and (7), we see that

$y = Dl$

Hence

$l = D^{-1}\,y$

Similarly, on pre-multiplying (5) by C^{-1}, we obtain

$$C^{-1}q = C^{-1\,l} + C^{-1}\,q\,A'\,i \qquad \textbf{(8)}$$

Under **industry technology** assumption, $A = BD$

Thus (8) becomes

$$C^{-1\,q} = C^{-1}\,l + C^{-1}\,q\,D'\,B'\,i$$

or

$$g = C^{-1}\,l + g\,(M')^{-1}\,q\,D'\,B'\,i$$
$$= C^{-1}\,l + g\,(D')^{-1}\,D'\,B'\,i$$
$$= C^{-1}\,l + g\,B'\,i \qquad \textbf{(9)}$$

On comparing (4) and (9), we see that

$y = C^{-1}\,l$

or $l = Cy$

Therefore under,

Commodity technology: $l = D^{-1}\,y$

Industry technology: $l = Cy$

Mixed Assumptions:

A2.20 For mixed assumption, the output matrix would be divided into two parts, such as

$$M = M_1 + M_2$$

Where M_2 is a matrix of those by-products which are to be transferred on the assumption of industry technology and the element of M_1 are outputs which, it seems reasonable to treat on the assumption of commodity technology. The formation of $M_1 + M_2$ involves splitting individual elements of M, since these elements may contain a mixture of products not all of which are to be treated in the same way.

A2.21 With this decomposition of M, we would have *commodity x commodity* table: $q = I (B R)q + f$

industry x industry table : $g = (R B) g + R f$

where $R = C^{-1}(I-D_2 ' i) + D_2$;

$D'_2 i$ denote a diagonal matrix formed from the vector of $D_2 i$

A2.22 Since R involves the matrix C_1^{-1}, the matrices C_1 and D_2 must be square matrices of the same order.

A2.23 It may further be mentioned that C_1 and D_2 are matrices similar to C and D except that they are now based on M_1 and M_2 respectively using q_1 vector in case of C_1, but q vector in case of D_2.

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